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OBSERVATION OF A NEW TYPE OF SOLITARY WAVES IN A ONE-DIMENSIONAL GRANULAR MEDIUM

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A new type of solitary waves has been found by numerical calculations of a string of particles interacting according to Hertz's law [1]. The author's analysis of the differential equation representing the long-wave approximation for this system also demonstrated the existence of steady-state solitary waves under definite conditions, consistent with the numerical calculations of a discrete string of particles. The nonlinear equation obtained in [1] is more general than the Korteweg–de Vries (KdV) equation, which includes nonlinear and dispersion effects in the first approximation for a broad class of physical systems [2].

The investigated system of particles has the distinctive feature that the law of interaction between them does not contain a linear component, even in the zeroth approximation. The given interaction of a one-dimensional string or for simple cubic packing corresponds to an equation of state of the medium (for uniaxial static compression) in the form

$$\sigma = E\varepsilon^{3/2}/[3(1-\nu^2)],$$

where σ is the stress, ε is the strain, and E and ν are the Young's modulus and Poisson ratio of the material of the particles. It is seen at once that the long-wave sound velocity in a medium described by this equation of state is

$$c_0^2 = \frac{1}{\rho}\frac{\partial\sigma}{\partial\varepsilon} = \frac{E\varepsilon^{1/2}}{2\rho(1-\nu^2)}$$

(ρ is the density of the medium). For $\varepsilon = 0$, i.e., in a string of particles not subjected to an external force, $c_0 = 0$. Consequently, the standard wave equation cannot be used to describe perturbations of arbitrary amplitude in the case of zero initial strain.

It is reasonable to except on the basis of the customary approach in this case, e.g., in impact created by a piston with any velocity in such a system, that a shock wave will be generated in it if dissipative processes are present. However, the numerical calculations in [1] have shown that when a triangular pulse acts on one end of the given string of particles, the initial disturbance decays into a train of solitary pulses.

For comparison with experiment it is more convenient to consider the given system driven by the impact of a piston of finite mass. The parameter determined in the numerical analysis and then compared with experiment in this case was the reaction to a rigid wall, on which

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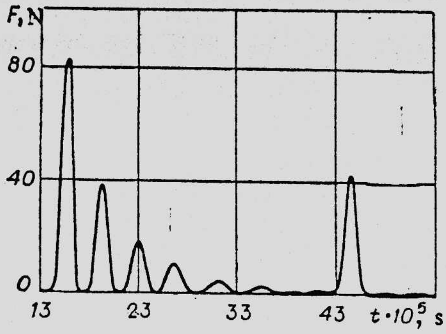

Fig. 1

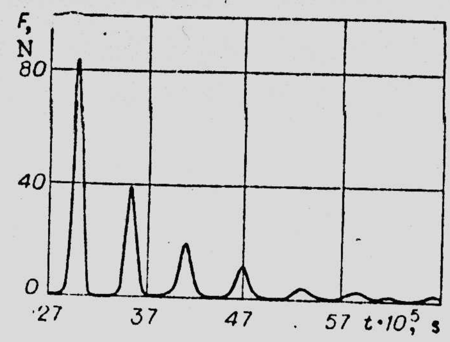

Fig. 2

rested the end of the string opposite to the impact-friven end. The interaction between the piston and the first particle and also between the last particle and the wall was chosen on the basis of Hertz's law. The energy and momentum in the numerical calculations were monitored within respective error limits of 10^{-2} and 10^{-5}%. An estimate of the relative error of determination of the particle velocities yields a value of the order of 10^{-2}%. The calculations were carried out according to a fourth-order Runge–Kutta scheme with a time step of $0.25 \cdot 10^{-5}$ sec. The mass of the piston was made equal to five times the mass of one particle. The constants in the interaction law were taken equal to $E = 2 \cdot 10^{11}$ N/m² and $\nu = 0.29$, corresponding to the properties of steels. The velocity of the piston was 0.5 m/sec, the particle diameter was $2R = 4.75 \cdot 10^{-3}$ m, and the density of the particle material was $\rho_0 = 7.8 \cdot 10^3$ kg/m³.

Figures 1 and 2 show the variation of the force F between a plane and the last particle in the cases of N = 20 and 40 particles, respectively. The start of interaction of the piston with the string of particles was taken as zero time. It is evident from Figs. 1 and 2 that the initial disturbance decays into a train of solitary waves. The occurrence of an isolated pulse at $t = 4.3 \cdot 10^{-4}$ s in Fig. 1 corresponds to the arrival of the reflected wave from the piston at the lower wall. The numerical calculations exhibited a steady-state behavior in the propagation of the generated solitary waves. An increase in the mass of the piston causes the number of these waves to increase.

An analysis shows that the dependence of the soliton phase velocity V on the particle velocity at the maximum, v_m, determined from the numerical calculations, coincides (within the error limits of the latter) with the dependence for a steady-state solitary wave described by the nonlinear equation derived in [1] (with the system subjected to an initial strain much smaller than the strain at the maximum of the soliton):

$$V = \left(\frac{2E}{(1-\nu^2)\,\pi\rho_0}\right)^{2/5}\left(\frac{16}{25}v_m\right)^{1/5}.$$

The given dependence $V(v_m)$ obviously differs from linear for a soliton described by the KdV equation and cannot be obtained from the latter. In our opinion, the existence of soliton solutions of the nonlinear equation derived in [1] for an indefinitely small but finite initial strain of the system is equivalent, from the physical point of view, to the possibility of solitary waves propagating in an unloaded string of particles.

TABLE 1

time interval and amplitude of pulses	N=20 experimental	N=20 numerical	N=40 experimental	N=40 numerical
τ_{12}, μs	35	37,5	48	55
τ_{13}, μs	72	72,5	91	110
τ_{14}, μs	105	110	135	170
F_1, N	71	83	52	85
F_2, N	40	39	24	39
F_3, N	21	19	18	20
F_4, N	10	11	11	11

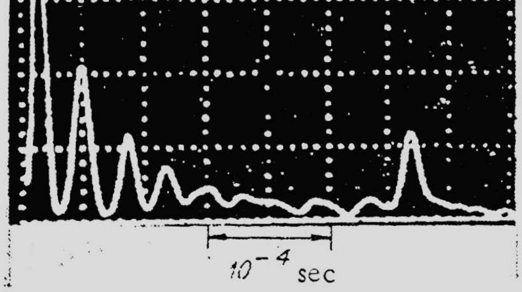

Fig. 3

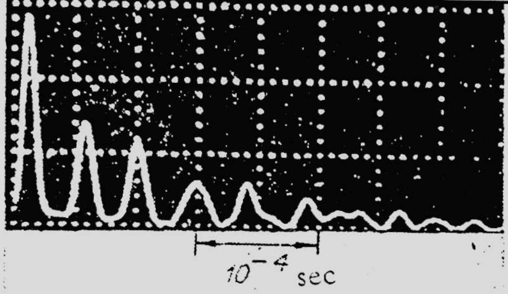

Fig. 4

The characteristic space scale of the solitary waves in the numerical calculations in · range of particle velocities satisfying the conditions for the validity of Hertz's law 0 m/sec) is practically independent of the amplitude of the disturbance and is equal to ;, i.e., a steady-state soliton is generated mainly by the motion of five particles. The ocity of the outermost particles is of the order of 1% of the velocity of the middle par-le. These properties depart significantly from the properties of solitons described by · KdV equation [2].

We have carried out experiments consistent with the above-described numerical treatment impact by a piston. A one-dimensional system of particles was created by the use of a rtz tube with an inside diameter of 5 mm, in which was placed a string of chrome steel h-15 balls (bearing type) with a diameter of 4.75 mm. The string rested on a disk of dened (HRC = 40) chrome-manganese-silicon steel 30KhGSA with a thickness of 2 mm, which not in contact with the quartz tube. A TsTS-19 (lead zirconate titanate PZT) wafer was ded to the underside of the disk by means of epoxy resin, serving as a piezoelectric nsducer of the force acting on the steel disk. The wafer, in turn, was attached to a ss rod with a length of 200 mm, which was heavily coated with epoxy resin and inserted in opper sheath. This configuration of the force sensor makes it possible to eliminate the luence of reflected waves in the rod; the latter condition was specially tested in radic experiments. The signal from the piezoelectric transducer was recorded by means a copper electrode soldered into the upper part of the PZT wafer, the underside of which bonded to the brass rod. The measurement arrangement (rC = 0.25 sec) was calibrated by acting a single ball and then comparing the measured symmetrical time dependence of the ess with the dependence calculated according to Hertz's law. The S8-17 oscilloscope used the given experiments operated in the internal-trigger mode, using the working signal.

Figures 3 and 4 show the experimental time dependence of the force acting on the lower k for N = 20 and 40 particles, respectively. The vertical scale is 18.3 N between large isions. The amplitudes and time parameters of the pulses, averaged over four experiments, shown in Table 1, along with the numerical data. The quantities τ_{12}, τ_{13}, τ_{14} are the time ervals between the maximum of the first soliton and the subsequent (as numbered by the ond index) maxima. The quantities F_1–F_4 are equal to the amplitudes of the pulses from first four solitons. The relative measurement error does not exceed 17% for the values F_1–F_4 and 7% for the time intervals τ_{12}, τ_{13}, τ_{14}.

It is evident from a comparison of Figs. 1 and 3 and the data of Table 1 that qualita-ve and quantitative agreement exists between the amplitudes of the solitary waves, their ber, and their time parameters for N = 20. For N = 40 the experimentally observed qualita-ve pattern is completely consistent with the numerical calculations (see Figs. 2 and 4), the amplitudes of the solitary waves (F_1 and F_2) in the experiments are much smaller than theoretical values (see Table 1) owing to the presence of dissipative processes, which are ored in the calculations. As a result, a large disparity is also observed in the time in-vals τ_{12}–τ_{14} in the given situation.

Thus, in the experiments we have observed a new type of solitary waves, which concur with numerical calculations of a discrete string of particles.

The given example is the only instance known to us where solitary waves are observed when propagation velocity of long-wave acoustic disturbances in the system is equal to zero. solitons in this case act as elementary steady-state excitations.

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LITERATURE CITED

1. V. F. Nesterenko, "Propagation of nonlinear compression pulses in granular media," Zh. Prikl. Mekh. Tekh. Fiz., No. 5 (1983).
2. G. B. Whitham, Linear and Nonlinear Waves, Wiley-Interscience, New York (1974).

EXPERIMENTAL ESTIMATION OF THE ULTIMATE STRAINS OF DYNAMICAL RUPTURE OF CYLINDRICAL SHELLS

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The question of the ultimate strains of cylindrical shells expanding under the action of detonation products (DP) of condensed explosives (CE) is discussed in [1-4]. Realization of the rupture criteria [2, 5] in application to rigidly plastic cylindrical shells is examined in [4].

The simplest estimation of the influence of a scale factor on the rupture radius of a shell loaded by a pressure pulse and storing some elastic energy to be expended entirely in rupture is presented in [6]. A modification of this approach that takes account of the dynamics of the shell loading process, its thickness, the plastic properties of the shell material, the type of CE loading, and the presence of tensile stress zones varying over the thickness as a shell expands under the action of DP, can be realized by applying the Taylor dependence [1]

$$y = \delta Y/p, \tag{1}$$

where y is the thickness of the stretched zone measured from the shell outer surface, δ is the shell running thickness, Y is the dynamic yield point of the shell material, $p = p_0(a_0/a)^{2k}$ is the running pressure on the shell inner surface, $p_0 = \rho_0 D^2/2(k + 1)$ is the pressure of instantaneous detonation, ρ_0 is the CE density, D is the detonation velocity, k is the DP isentropy index, and a_0 and a are the initial and running inner radii of the shell.

Long cylindrical shells ($L_0/a_0 > 12$, L_0 is the shell length) whose internal cavity is filled with a CE charge initiated at the point of endface surface symmetry were subjected to explosive loading. By use of a high-speed optical device, values of the relative outer fracture radius b_f/b_0 (b_0 is the initial outer radius of the shell, and b_f is the rupture radius) found by the outbreak of DP on the outer surface, were determined. Certain results of the experiment are presented in Fig. 1 for medium carbon steel at successive times 1) t = 11.2; 2) 22.4; 3) 28.8; 4) 35.2 μsec.

We shall consider the development of rupture to be the propagation of a separation crack. Since only the tensile zone that expends the stored energy on rupture can be subjected to unloading, the store of this energy equals the quantity

$$E_1 = A\varepsilon_1 \frac{2b - y}{2} yL_0, \tag{2}$$

where ε_1 is the specific volume energy in the stretched zone expended on rupture (the development of the separation crack), $(2b - y)/2 = R_y$ is the mean radius of the tensile zone, and A = const.

Taking account of (1), the dependence (2) has the form

$$E_1 = A\frac{Y}{p}\delta b L_0\left(1 - \frac{y}{2b}\right)\varepsilon_1.$$

Moreover, $\varepsilon_1 \sim Y^2/E$, and $Y \sim (\varepsilon_1 E)^{1/2}$, i.e., $\varepsilon_1 Y \sim \varepsilon_1^{3/2} E^{1/2} = B$, and then

$$E_1 = \frac{C}{p}\delta b L_0\left(1 - \frac{y}{2b}\right), \tag{3}$$

where C = AB = const.

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